**Important** Attach a hardcopy of all MATLAB code.

1) As done in class, the zero-DC square wave function, with period \( T = 1 \), is

\[
x(t) = -\frac{1}{2} + u(t + \frac{1}{4}) - u(t - \frac{1}{4}) \quad \text{for} \ |t| < \frac{1}{2},
\]

extended periodically. Its Fourier expansion,

\[
x(t) = \sum_{k=-\infty}^{\infty} a_k e^{2\pi jkt} \quad [\text{since the fundamental frequency} \ \omega_0 = 2\pi \ (\text{why}?)]
\]
determines the Fourier series \( a_k \).

Consider the \( N \)th approximation to its Fourier expansion, \( x_N(t) = \sum_{k=-N}^{N} a_k e^{2\pi jkt} \):

a) By hand, plot \( x(t) \) for more than one period, around \( t = 0 \), and evaluate the maximum, minimum and mean.

b) Either by calculating directly, or by using your class notes for \( a_k \), simplify its Fourier series \( a_k \) using \( T = 1 \) and \( \omega_0 = 2\pi \).

c) Using MATLAB, plot \( x_N(t) \) for \( N = 4 \) over the same range as (a). Estimate the maximum, minimum and mean. The plot should resemble the plot in (a)—how good is it?

d) Repeat (c) for \( N = 16 \).

e) Repeat (c) for \( N = 128 \).

**Optional Hint** Because \( x(t) \) is even, real, and has zero DC, the calculation may be simplified by using

\[
a_k e^{2\pi jkt} + a_{-k} e^{-2\pi jkt} = a_k \left( e^{2\pi jkt} + e^{-2\pi jkt} \right) = 2a_k \cos(2\pi kt),
\]
giving \( x_N(t) = \sum_{k=1}^{N} 2a_k \cos(2\pi kt) \) allowing you to avoid complex numbers in your program.

2) Consider the discrete periodic signal \( x[n] = 1 + \sin(2\pi \frac{n}{8}) \):

a) Plot \( x[n] \) for the range \([0,7]\).

b) Compute its Fourier series coefficients \( a_k \) for the ranges of \( k : [0,7], [-3,4], \) and \([8,15]\).

c) Use MATLAB to help solve this problem. Use the following commands:

```matlab
n = [0:7];
x = 1+sin(2*pi*n/8);
A = fft(x)
```
How is \( A \) related to \( a_k \)? There may be an overall constant factor disagreement. When comparing the elements of the array \( A \) to \( a_k \), which value of \( k \) corresponds to the first element of \( A \) (i.e., \( A(1) \))? 

3) Consider the RLC circuit pictured in Figure P3.20 (page 254) in Oppenheim & Willsky, except that R, L, and C are unknown (i.e. keep them as “R”, “L”, and “C”).
   a) Using an input signal of \( V_0 \exp(j\omega t) \), and knowing that because the RLC circuit is an LTI system the output signal will be \( H(j\omega)V_0\exp(j\omega t) \), derive an expression for the transfer function \( H(j\omega) \).
   b) For the values of R, L, and C in the book \( (R = 1\Omega, L = 1H, C = 1F) \), calculate \( |H(j\omega)| \): the magnitude of \( H(j\omega) \). Sketch it (by hand) for \( 0 \leq \omega < 3 \) (i.e. Make sure the beginning and end values are right and get the general trend in between). Is this a low-pass, band-pass, or high-pass filter?

4) For each of the following signals \( x(t) \), where \( a \) and \( L \) are unknown constants, sketch the signal and compute its Fourier Transform \( X(j\omega) \):
   a) \( x(t) = a(u(t) - u(t - L)) \)
   b) \( x(t) = a(u(t + L) - u(t)) \)
   c) \( x(t) = a(u(t + L/2) - u(t - L/2)) \)
   d) \( x(t) = au(t + L) + au(t + L/2) - au(t - L/2) - au(t - L) \)

Note For the next problem, you may find this relation useful:
\[
\int_{-\infty}^{\infty} \exp(jbx)\exp(-a^2x^2)dx = 2\int_{0}^{\infty} \cos(bx)\exp(-a^2x^2)dx = \frac{\sqrt{\pi}}{a} \exp(-b^2/4a^2)
\]

5) Consider the Gaussian signal \( x(t) = \exp\left(-t^2/2\sigma^2\right) \). [\( \sigma^2 \) is called the variance and parameterizes the width of the bell curve.]
   a) Sketch \( x(t) \) for the range \([-2, +2]\) if \( \sigma = 2 \).
   b) Compute its Fourier Transform \( X(j\omega) \).
   c) How does \( |X(j\omega)| \) fall off as \( |\omega| \to \infty \)?
   d) Where is the maximum of \( |X(j\omega)| \)? How would this change if the signal was modified to be the related signal \( x(t) = \exp(j\omega_0t - t^2/2\sigma^2) \)?