All plots should be done by hand, not by computer (a calculator, if needed, is OK).

1) Consider the following systems, where \( x \rightarrow [\text{System}] \rightarrow y \) (these are the same as from last week)
   a) \( y(t) = \cosh(x(t)) = \frac{1}{2}(e^{x(t)} + e^{-x(t)}) \)
   b) \( y[n] = \text{Run}_{-\infty}^{n} x[n] = \sum_{n=-\infty}^{n} x[n'] \)
   c) \( y[n] = |x[n+1] - x[n]| \)
   d) \( \frac{d}{dt}y(t) + \omega y(t) = \omega^2 x(t) \)
   i) Which of the systems (a-d) are time invariant?
   ii) Which of the systems (a-d) are linear?
   iii) In a few words and/or equations, explain why system (b) is or isn’t time invariant
   iv) In a few words and/or equations, explain why system (c) is or isn’t time invariant
   v) In a few words and/or equations, explain why system (d) is or isn’t time invariant
   vi) In a few words and/or equations, explain why system (a) is or isn’t linear
   vii) In a few words and/or equations, explain why system (b) is or isn’t linear
   viii) In a few words and/or equations, explain why system (c) is or isn’t linear

2) For each of the following discrete LTI systems compute, and simplify, the output signal \( y[n] \) when the input signal \( x[n] \) is an unknown signal \( f[n] \)
   a) Impulse Response \( h[n] = \delta[n - n_0] \); also, describe in a few words the effect of the system
   b) Impulse Response \( h[n] = \delta[n] - \delta[n - 1] \), the signed edge-detector
   c) Impulse Response \( h[n] = -\delta[n + 1] + 2\delta[n] - \delta[n - 1] \), the signed curvature-detector
   d) Impulse Response \( h[n] = (\delta[n + 1] + 2\delta[n] + \delta[n - 1])/4 \), a symmetric blurring function
   e) Which of these four systems are causal, and why?

3) Let an input signal \( x[n] \) be defined as \( x[n] = \frac{\cos(\pi n/4)}{1 + |n|} \). Using this input signal, you will plot the output signal for different systems.
a) First, plot the original signal $x[n]$ in the range $[-6, +6]$.
b) Now, using the discrete LTI system with Impulse Response $h[n] = \delta[n] - \delta[n - 1]$ (the signed edge-detector system), plot the output $y[n]$ in the range $[-6, +6]$. It should be most extreme where $x[n]$ is changing fastest.
c) Similarly, using the discrete LTI system with Impulse Response $h[n] = -\delta[n + 1] + 2\delta[n] - \delta[n - 1]$ (the signed curvature-detector system), plot the new output $y[n]$ in the range $[-6, +6]$. It should be most extreme where $x[n]$ has the strongest curvature.

For the problems below, we use the notation that $y$ is the output of a system for an input $x$.

4) Consider the discrete Linear Time Invariant (LTI) system with Impulse Response $h[n] = u[n]$:
   a) For $x[n] = u[n]$, compute $y[n]$
   b) For $x[n] = u[n]\alpha^n$, $|\alpha| < 1$, compute $y[n]$

5) Consider the following continuous LTI systems with different Impulse Responses, when given a common input signal $x(t) = u(t)$:
   a) For $h(t) = u(t)$, compute $y(t)$ and simplify.
   b) For $h(t) = u(t)e^{-bt}$, $b > 0$, compute $y(t)$ and simplify.
   c) For $h(t) = u(t)e^{j\omega t}$, compute $y(t)$ and simplify.
   d) For $h(t) = u(t)\cos(\omega t)$, compute $y(t)$ and simplify.

6) Consider the continuous LTI system with Impulse Response $h(t) = u(t)e^{-at}$, $a > 0$:
   a) For $x(t) = u(t)$, compute $y(t)$ and simplify.
   b) For $x(t) = u(t)e^{-at}$, compute $y(t)$ and simplify.
   c) For $x(t) = u(t)e^{j\omega t}$, compute $y(t)$ and simplify.

Useful identities:
\[
\cos(x) = \frac{(e^{ix} + e^{-ix})}{2}
\]
\[
\sin(x) = \frac{(e^{ix} - e^{-ix})}{2j}
\]