Optimum dilution rate for cell productivity with substrate inhibition.
Instructor: Nam Sun Wang

Operating condition:
\[ s_f := 200 \]

Model parameters:
\[ \mu_m := 0.3 \quad K := 50 \quad K_i := 0.01 \quad A := 0.004 \quad B := 0.001 \]
\[ \mu(s) := \frac{\mu_m s}{K + s + K_i s^2} \quad Y(s) := A + B s \]

Maximum specific growth rate \( \mu \): \( s := 10 \) ... initial guess

Given \( \frac{d}{ds} \mu(s) = 0 \) \( s := \text{Find}(s) \) \( s = 70.711 \)
\( \mu(s) = 0.124 \)

An initial guess of \( s=0 \) is o.k. in Mathcad v5, but no good in v7.

Note that the "root" function does not seem to be as accurate:
\[ s := 0 \quad s := \text{root} \left( \frac{d}{ds} \mu(s), s \right) \]
\( s = 51.301 \quad \mu(s) = 0.121 \)

Initial guesses:
\[ x := 1 \quad s := 0 \]

Given Steady-state equations:
\[ \frac{dx}{dt} = 0 = (\mu(s) - D) \cdot x \]
\[ \frac{ds}{dt} = 0 = D \cdot (s_f - s) - \frac{1}{Y(s)} \cdot \mu(s) \cdot x \]
\[ \text{ans}(D) := \text{Find}(x, s) \quad x(D) := \text{ans}(D) \]

An example: \( \text{ans}(0.1) = \begin{pmatrix} 5.683 \\ 29.289 \end{pmatrix} \)

Dependence of \( D \cdot x \) on \( D \):
\( D := 0.01, 0.02, 0.124 \)

Note that cell productivity increases monotonically with \( D \), right up to maximum \( \mu \), at which point cell washout occurs. The maximum cell productivity occurs at: \( D := 0.124 \)

The maximum cell productivity is: \( D \cdot x(D) = 1.155 \)