Solve a set of ordinary differential equations (ODEs) with given initial conditions. The solution looks quite different for different initial conditions and model parameters. Experiment!

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Let's solve the predator-prey example described by the following set of ODEs.

\[
\frac{d}{dt} \text{rabbit}(\text{rabbit, fox}) = 2 \cdot \text{rabbit} - 2 \cdot \text{rabbit} \cdot \text{fox} \\
\frac{d}{dt} \text{fox}(\text{rabbit, fox}) = -\text{fox} + \text{rabbit} \cdot \text{fox}
\]

Specify ODEs which describe the rate of change and initial conditions.

\[
\text{rate}_\text{rabbit}(\text{rabbit, fox}) := 2 \cdot \text{rabbit} - 2 \cdot \text{rabbit} \cdot \text{fox} \quad \text{I.C.: rabbit} := 3 \\
\text{rate}_\text{fox}(\text{rabbit, fox}) = -\text{fox} + \text{rabbit} \cdot \text{fox} \quad \text{I.C.: fox} := 2
\]

Combine the above two coupled ODEs into a vector.

\[
\frac{dy}{dt}(t, y) := \begin{pmatrix}
\text{rate}_\text{rabbit}(y_0, y_1) \\
\text{rate}_\text{fox}(y_0, y_1)
\end{pmatrix} \quad \text{I.C.: } y_\text{initial} := \begin{pmatrix}
\text{rabbit} \\
\text{fox}
\end{pmatrix}
\]

Numerically integrate with the Runge-Kutta's method from \(t=0\) to \(10\) at 100 fixed intervals.

\[
\text{ty} := \text{rkfixed}(y_\text{initial}, 0, 10, 100, \text{dydt})
\]

The output is stored in a matrix whose 0th column is the independent variable (time), 1st column is the 1st variable (rabbit), and 2nd column is the 2nd variable (fox).

The time trajectories and phase plane plot are shown in the images.

Time Trajectories for rabbits and foxes

Phase Plane Plot