Dynamic simulation of predator-prey kinetics with the Euler's method.
Instructor: Nam Sun Wang

Assign model variables:
\[ \alpha := 0.3 \]
\[ \beta := 0.01111 \]
\[ \gamma := 0.2106 \]
\[ \delta := 0.0002632 \]

Specify initial conditions:
\[ y_{0,0} := 700 \quad \text{... prey population} \]
\[ y_{1,0} := 22 \quad \text{... predator population} \]

Number of points and index variable.
\[ N := 500 \quad \text{... number of points} \]
\[ i := 0..N \quad \text{... index variable} \]

Divide time into equally spaced intervals.
\[ t_{\text{max}} := 100 \quad \text{... range of time} \]
\[ \delta t := \frac{t_{\text{max}}}{N} \quad \text{... step size} \]
\[ t_i := i \cdot \delta t \quad \text{... assign time} \]

Dynamic equations. Solve the coupled set of ODEs with the Euler's method in N steps.
Note: coupled equations must be grouped together in a vector.

\[
\begin{align*}
\begin{bmatrix}
    y_{0,i+1} \\
    y_{1,i+1}
\end{bmatrix} &=
\begin{bmatrix}
    y_{0,i} + (\alpha \cdot y_{0,i} - \beta \cdot y_{0,i} \cdot y_{1,i}) \cdot \delta t \\
    y_{1,i} + (-\gamma \cdot y_{1,i} + \delta \cdot y_{0,i} \cdot y_{1,i}) \cdot \delta t
\end{bmatrix} \\
& \quad \quad \text{Prey eqn} \\
& \quad \quad \text{Predator eqn}
\end{align*}
\]

Plots of state variables

![Phase Diagram](image1)

![Phase Diagram](image2)

Prey & Predator Populations

- Prey Population
- Predator Population
General formulation of Euler's method for solving $\frac{dy}{dx}=f(x,y)$ with a step size of $h$.

The following one line formulation is valid for a multidimensional dynamic system. It is most convenient to employ a vector notation, in which case the whole Euler's algorithm is only one single line. (Do not be confused by the notation -- from the description "$\frac{dy}{dx}$", it is clear that $x$ is the independent (scalar) variable, and $y$ is the dependent (vector) variable.)

$$\text{Euler}(x,y,f,h) := y + f(x,y) \cdot h$$

Apply the Euler's method to the predator-prey dynamic system.

$$\frac{dy}{dt}(t, y) := \begin{cases} \alpha y_0 - \beta y_0 \cdot y_1 \\ \gamma y_1 + \delta y_0 \cdot y_1 \end{cases}$$

I.C. \quad y^{<0>} := \begin{pmatrix} 700 \\ 22 \end{pmatrix} \quad \ldots \text{prey (}y_0\text{) at } t=0

\ldots \text{ predator (}y_1\text{) at } t=0

\text{Integration parameters:}

Integration steps: $i := 0 \ldots N$ \quad Step size: $h := 0.2$ \quad Time: $t_i := i \cdot h$

Integration (update step by step) with Euler's method.

$$y^{<i+1>} := \text{Euler}(t_i, y^{<i>}, \frac{dy}{dt}, h)$$

The 0th row of $y$ contains the 0th variable (prey); the 1st row of $y$ is the 1st variable (predator).

$$\text{prey} := \left(y^{<0>}\right)^T \quad \text{predator} := \left(y^{<1>}\right)^T$$

Plot the result.