1) Consider a causal impulse response $h[n]$ defined implicitly by $y[n] + a_1 y[n-1] = b_0 x[n] + b_1 x[n-1]$, where $a_1 = 2, b_0 = 1, b_1 = -5$.

a) Evaluate the transfer function $H(z)$ and its ROC.

b) Justify the statement that $H(z)$ is not stable and does not have a stable inverse.

c) Find a causal transfer function $H_{\text{min}}(z)$ and its ROC such that $|H_{\text{min}}(e^{j\omega})| = |H(e^{j\omega})|$ but with $H_{\text{min}}(z)$ minimum phase (i.e. stable with a stable inverse). Be careful to find the correct overall gain factor.

d) Consider the causal impulse response $h_{\text{min}}[n]$ which is the inverse Z transform of $H_{\text{min}}(z)$. Without calculating $h_{\text{min}}[n]$ explicitly, find the constants $\tilde{a}_1, \tilde{b}_0, \tilde{b}_1$ so that $h_{\text{min}}[n]$ is defined implicitly by $y[n] + \tilde{a}_1 y[n-1] = \tilde{b}_0 x[n] + \tilde{b}_1 x[n-1]$.

e) One reason to compute $H_{\text{min}}(z)$ is to undo the distortion gain of $H(z)$. For this we need $H_{\text{min}}^I(z)$, the inverse of $H_{\text{min}}(z)$. Evaluate $H_{\text{min}}^I(z)$. For $H_{\text{min}}^I(z)$ causal, is it stable?

f) What is the magnitude of the transfer function of the total system defined by applying $H(e^{j\omega})$ and then following it with $H_{\text{min}}^I(e^{j\omega})$? Simplify.

2) For each of the following causal, allpass filters, answer the subsequent questions. Feel free to use Matlab.

a) $H(z) = \frac{z^{-1} - a^*}{1 - a z^{-1}}$, where $a = 1/8$

b) $H(z) = \frac{z^{-1} - a^*}{1 - a z^{-1}}$, where $a = -1/8$

c) $H(z) = \frac{z^{-1} - a^*}{1 - a z^{-1}} \frac{z^{-1} - a}{1 - a^* z^{-1}}$, where $a = j/8$

i) Plot the gain $|H(e^{j\omega})|$ in dB for $\omega$ in the range $[0, 2\pi]$.

ii) Plot the phase in degrees for $\omega$ in the range $[0, 2\pi]$.

iii) Plot the unwrapped phase in degrees for $\omega$ in the range $[0, 2\pi]$. Is this consistent with the answers in (ii)? Why?

iv) Plot the group delay (in samples) for $\omega$ in the range $[0, 2\pi]$. Is it always positive?
3) Consider the FIR system described by \( H(z) = (1 - az^{-1})(1 - a^* z^{-1}) \) for \( a = -4j \).


b) Plot the gain \( |H(e^{j\omega})| \) in dB for \( \omega \) in the range \([0, 2\pi]\).

c) Find a compensating filter \( H_c(z) \) such that \( |H(e^{j\omega})H_c(e^{j\omega})| = 1 \) and \( H_c(z) \) is stable. Hint: Decompose \( H \) into minimum phase and allpass components, and invert the minimum phase component.