Useful identities: \[ \cos(x) = (e^{ix} + e^{-ix})/2 \]
\[ \sin(x) = (e^{ix} - e^{-ix})/2i \]
\[ f(|s|) = u(s)f(s) + u(-s)f(-s) \]

Also, feel free to use properties of convolution (e.g. associativity) whenever it helps.

1) Compute and simplify:
   a) \( e^{j\omega t} * (e^{-bt} u(t)) \), \( b > 0 \)
   b) \( (e^{-at} u(t)) * (e^{-bt} u(t)) \), \( a,b > 0, a \neq b \)
   c) \( e^{j\omega t} *( (e^{-at} u(t)) * (e^{-bt} u(t)) ) \), \( a,b > 0, a \neq b \)
   d) \( (e^{j\omega t} * (e^{-at} u(t))) * (e^{-bt} u(t)) \), \( a,b > 0, a \neq b \)
   e) \( ((e^{-bt} u(t)) * e^{j\omega t}) * (e^{-at} u(t)) \), \( a,b > 0, a \neq b \)

2) Compute and simplify:
   a) \( e^{-a|t|} \sin(\omega s), a > 0 \)
   b) \( e^{-a|t|} \cos(\omega s), a > 0 \)
   c) \( e^{-a|t|} \cos(\omega s) * e^{-b|t|}, a,b > 0 \)

3) Consider the continuous LTI system with \( y(s) = x(s - s_0) / 2 \):
   a) Compute \( h(s) \)
   b) Compute its inverse impulse response \( h_I(s) \)
   c) Demonstrate that \( h_I(s) * h(s) = \delta(s) \)

4) Consider the continuous LTI system with \( y(t) = -\int_{-\infty}^{t} x(t') dt' \):
   a) Compute \( h(t) \)
   b) Compute its inverse impulse response \( h_I(t) \)
   c) Demonstrate that \( h_I(t) * h(t) = \delta(t) \)

5) Consider the continuous LTI system with \( y[n] = \beta \text{Diff}\{x[n]\} = \beta x[n] - \beta x[n-1], \beta \neq 0 \):
   a) Compute \( h[n] \)
   b) Compute its inverse impulse response \( h_I[n] \)
c) Demonstrate that $h_i[n] * h[n] = \delta[n]

6) Consider the continuous LTI system with

\[ y[n] = \text{Diff}\{\text{Diff}\{x[n]\}\} = \text{Diff}\{x[n] - x[n-1]\} = x[n] - 2x[n-1] + x[n-2] \]

a) Compute $h[n]$

b) Prove that its inverse impulse response is given by $h_i[n] = (n + 1)u[n]$ by showing that $h_i[n] * h[n] = \delta[n]$ (you may want to use $(n + 1)u[n] = u[n] * u[n]$)

7) Prove that each of the following are either stable or unstable systems:

a) \[ y(t) = -x(t - t_0) = \int_{-\infty}^{\infty} x(t')\left(-\delta((t - t') - t_0)\right)dt' \]

b) \[ y(t) = \int_{-\infty}^{\infty} x(t')e^{-b(t-t')}u(t - t')dt', b > 0 \]

c) \[ y(t) = \int_{-\infty}^{\infty} x(t')e^{-b(t-t')} dt', b > 0 \]

d) \[ y(s) = \int_{-\infty}^{\infty} x(s')e^{-b(s-s')}ds', b > 0 \]

e) \[ y[n] = \sum_{k=-\infty}^{\infty} x[k]\alpha^{-(n-k)}, |\alpha| < 1 \]

f) \[ y[n] = \sum_{k=-\infty}^{\infty} x[k]\alpha^{-(n-k)}, |\alpha| < 1 \]

8) Consider the causal system defined by a circuit with input and output satisfying the differential equation \[ \ddot{y}(t) + 3\dot{y}(t) + 2y(t) = x(t) \].

a) Find $y(t)$ for $x(t) = \delta(t)$ (i.e. the impulse response).

b) Find $y(t)$ for $x(t) = u(t)$

Note: The homogeneous solution, i.e., the most general solution to this differential equation when $x(t) = 0$, is given by $y_h(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$ where $c_1$ and $c_2$ are unknown constants and $\lambda_1$ and $\lambda_2$ are the two distinct solutions to the system’s characteristic equation $\lambda^2 + 3\lambda + 2 = 0$. This is important since it will also be the solution to part (a) for positive $t$.

9) For each causal system find the impulse response $h[n]$: 

a) \[ y[n] = (x[n] + x[n-1] + x[n-2]) / 3 \]

b) \[ y[n] = c\alpha x[n] + \beta y[n-1], |\beta| < 1 \]

c) \[ y[n] = x[n] + \beta y[n-2], |\beta| < 1 \]

d) \[ y[n] = x[n] + x[n-1] + \beta y[n-2], |\beta| < 1 \]