All plots should be done by hand, not by computer (a calculator, if needed, is OK).

1) Consider the following systems, where \( x \rightarrow \text{[System]} \rightarrow y \)

a) \( y(t) = \cosh(x(t)) = \frac{1}{2}(e^{x(t)} + e^{-x(t)}) \)

b) \( y[n] = \text{Run}_{-\infty}^{n}x[n] = \sum_{n=-\infty}^{n} x[n'] \)

c) \( y[n] = |x[n+1] - x[n]| \)

d) \( \frac{d}{dt}y(t) + \omega y(t) = \omega^2 tx(t) \)

a) Which of the systems (a-c) are instantaneous/ultralocal/memoryless?

b) Which of the systems (a-c) are causal?

c) Which of the systems (a-c) are invertible?

d) Which of the systems (a-d) are time invariant?

e) Which of the systems (a-d) are linear?

f) In a few words and/or equations, explain why system (a) is or isn’t instantaneous/ultralocal/memoryless

h) In a few words and/or equations, explain why system (b) is or isn’t causal

i) In a few words and/or equations, explain why system (b) is or isn’t invertible

j) In a few words and/or equations, explain why system (c) is or isn’t invertible

k) In a few words and/or equations, explain why system (d) is or isn’t invertible

l) In a few words and/or equations, explain why system (b) is or isn’t causal

m) In a few words and/or equations, explain why system (c) is or isn’t causal

n) In a few words and/or equations, explain why system (d) is or isn’t time invariant

o) In a few words and/or equations, explain why system (a) is or isn’t linear

p) In a few words and/or equations, explain why system (b) is or isn’t linear

q) In a few words and/or equations, explain why system (c) is or isn’t linear

r) In a few words and/or equations, explain why system (d) is or isn’t linear

2) For each of the following discrete LTI systems compute, and simplify, the output signal \( y[n] \) when the input signal \( x[n] \) is an unknown signal \( f[n] \)

a) Impulse Response \( h[n] = \delta[n - n_0] \); also, describe in a few words the effect of the system
b) Impulse Response \( h[n] = \delta[n] - \delta[n - 1] \), the signed edge-detector

c) Impulse Response \( h[n] = -\delta[n + 1] + 2\delta[n] - \delta[n - 1] \), the signed curvature-detector

d) Impulse Response \( h[n] = (\delta[n + 1] + 2\delta[n] + \delta[n - 1])/4 \), a symmetric blurring function

e) Which of these four systems are causal, and why?

3) Let an input signal \( x[n] \) be defined as \( x[n] = \frac{\cos(\pi n/4)}{1 + |n|} \). Using this input signal, you will plot the output signal for different systems.

a) First, plot the original signal \( x[n] \) in the range [-6, +6].

b) Now, using the discrete LTI system with Impulse Response \( h[n] = \delta[n] - \delta[n - 1] \) (the signed edge-detector system), plot the output \( y[n] \) in the range [-6, +6]. It should be most extreme where \( x[n] \) is changing fastest.

c) Similarly, using the discrete LTI system with Impulse Response \( h[n] = -\delta[n + 1] + 2\delta[n] - \delta[n - 1] \) (the signed curvature-detector system), plot the new output \( y[n] \) in the range [-6, +6]. It should be most extreme where \( x[n] \) has the strongest curvature.